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AN ANALYSIS OF HOLDUP IN HORIZONTAL TWO-PHASE GAS-LIQUID FLOW

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Abstract—The Butterworth form of correlation for holdup in two-phase gas–liquid flow is justified theoretically for certain conditions. In addition, a wide range of experimental data were used to show that holdup data may be broadly classified into three major groups based on the flow pattern, and different relationships were found to represent the data in each group. Thus for slug and plug flow, the holdup is given by the Armand type of equation; for stratified flow the holdup is given by the theoretical equations which are derived while annular flow data are satisfactorily represented by a semi-empirical correlation.

INTRODUCTION

Butterworth (1975) surveyed the literature on two-phase holdup and suggested by intuitive reasoning that a number of the more commonly used holdup prediction equations may be represented by the relation,

$$\left[\frac{\bar{R}_L}{\bar{R}_G}\right] = A\left[\frac{1-x}{x}\right]^p\left[\frac{\rho_G}{\rho_L}\right]^q\left[\frac{\mu_L}{\mu_G}\right]^r \qquad [1]$$

where $\bar{R}$ is the holdup, x the dryness fraction, ρ the density and μ the absolute viscosity, and the subscripts L and G refer to the liquid phase and gas phase respectively. The factors A, p, q and r, were shown to assume varying numerical values depending on which particular model was under consideration. Thus [1] provided a link for the various suggested holdup relations which, in their original forms, not only appeared unrelated, but sometimes gave conflicting results. However, what was lacking was some theoretical basis for the form of equation given by [1], and some rational explanation as to why there existed such a wide variation in the numerical values of the factors A, p, q and r with the various models that have been proposed.

Spedding & Chen (1979a, 1979b) in deriving holdup equations for the cases of ideal stratified and ideal annular horizontal flows, have shown that the form of [1] may in fact be analytically derived for certain situations. In these cases, it was found that the values of A, p, q and r varied with the ideal flow patterns considered, the flow regimes, namely laminar or turbulent, and also with the range of the holdup values. It is the purpose of this work to expand the development to cover a wider range of application.

THEORETICAL DEVELOPMENT

Ideal equilibrium stratified flow and annular flow are depicted schematically in figures 1 and 2 respectively. For stratified flow, by taking a force balance in the liquid phase and in the gas phase separately, it is possible to write,

$$-A_L\left[\frac{dp}{dl}\right]_{LF} - \tau_{WL}S_L + \tau_i S_i = 0 \qquad [2]$$

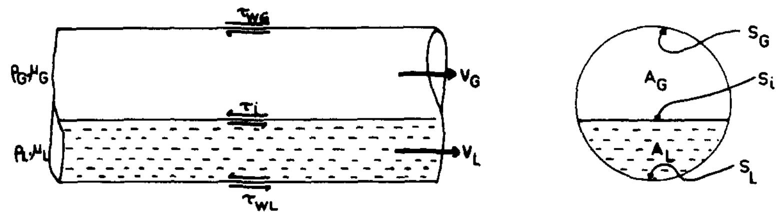

Figure 1. A schematic diagram of ideal stratified flow.

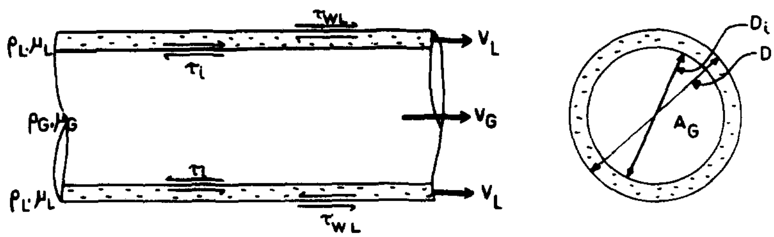

Figure 2. A schematic diagram of ideal annular flow.

$$-A_G\left[\frac{\mathrm{d}p}{\mathrm{d}l}\right]_{GF} - \tau_{WG}S_G + \tau_i S_i = 0) \qquad [3]$$

and for annular flow,

$$-A_G\left[\frac{\mathrm{d}p}{\mathrm{d}l}\right]_{GF} - \tau_i S_i = 0 \qquad [4]$$

$$-A_L\left[\frac{\mathrm{d}p}{\mathrm{d}l}\right]_{LF} + \tau_i S_i - \tau_{WL}S_L = 0 \qquad [5]$$

where A is the flow area for the particular phase, $[\mathrm{d}p/\mathrm{d}l]$ the pressure gradient, τ the shear stress, S the length in a cross section where shear forces are experienced. The subscript F refers to the frictional component, while G and L refer to the gas and liquid phases, i refers to the interface, and W refers to the wall. It is assumed that the pressure gradients in the liquid and the gas phases are equal

$$\left[\frac{\mathrm{d}p}{\mathrm{d}l}\right]_{LF} = \left[\frac{\mathrm{d}p}{\mathrm{d}l}\right]_{GF} \qquad [6]$$

in a similar manner to that suggested by Taital & Dukler (1976). The shear stresses may be evaluated as in the case of single phase flow,

$$\tau_{WL} = f_L \frac{\rho_L \bar{V}_L^2}{2} \qquad [7]$$

$$\tau_{WG} = f_G \frac{\rho_G \bar{V}_G^2}{2} \qquad [8]$$

$$\tau_i = f_i \rho_G \frac{(\bar{V}_G - \bar{V}_i)^2}{2} \qquad [9]$$

where $\bar{V}$ is the actual average velocity, and f the friction factor which may be expressed in the Blasius form for smooth pipes.

$$f_L = C_L\left[\frac{\bar{D}_L\bar{V}_L\rho_L}{\mu_L}\right]^{-m_L} \quad [10]$$

$$f_G = C_G\left[\frac{\bar{D}_G\bar{V}_G\bar{\rho}_G}{\mu_G}\right]^{-m_G}. \quad [11]$$

By combining [4] and [5],

$$\frac{\tau_i S_i}{A_G} = \frac{\tau_{WL}S_L}{A_L} - \frac{\tau_i S_i}{A_L}. \quad [12]$$

Since $S_i = \pi D_i$ and $S_L = \pi D$, and by using the form of the shear stresses in [7] and [9], then [12] reduces to

$$[f_i\rho_G\bar{V}_G^2 D_i]/\bar{R}_G = f_L\rho_L\bar{V}_L^2 D \quad [13]$$

which may be written in the form,

$$\bar{R}_L = \frac{1}{1+\left[\frac{f_i}{f_L(1-\bar{R}_L)^{1/2}}\right]^{1/2}\left[\frac{\rho_G}{\rho_L}\right]^{1/2}\left[\frac{Q_G}{Q_L}\right]} \quad [14]$$

where, for a smooth pipe, C is a numerical constant, assuming the value of 16 or 0.046 depending on whether the flow is laminar or turbulent, and m also assuming the values of 1 or 0.2 correspondingly. $\bar{D}$ is the hydraulic diameter for the phase, being four times the actual flow area over the wetted perimeter.

It may be assumed that $\bar{V}_G \gg \bar{V}_i$ (Bergelin & Gazley 1949) and that for a smooth interface, $f_i \simeq f_G$. The above equations were solved by using numerical methods and curve fitting techniques along the lines of those employed by Spedding & Chen (1979b) to give a series of equations which depended on flow pattern and regime as well as the range of holdup values.

However, in the case of annular flow the assumption of $f_i = f_G$ is well-known to be inappropriate. The annular liquid film is supported by a rather complicated system of forces and the liquid surface is always covered with various types of waves (Butterworth 1972). Wallis (1970) has shown that for such cases the friction factor may be approximated by

$$f_i = 0.005(1+75\bar{R}_L). \quad [15]$$

The use of [15] in evaluation [14] might perhaps give a better representation of the true situation in annular flow.

RESULTS

Stratified flow

With horizontal equilibrium stratified flow, for turbulent–turbulent and laminar–laminar gas–liquid flow, the form of [1] was expressed in terms of the volumetric flow rate, Q, instead of the dryness fraction,

$$\left[\frac{\bar{R}_G}{\bar{R}_L}\right] = K\left[\frac{Q_G}{Q_L}\right]^a\left[\frac{\rho_G}{\rho_L}\right]^b\left[\frac{\mu_G}{\mu_L}\right]^c \quad [16]$$

thus it became more amenable to subsequent interpretation. Therefore [16] will be used throughout this work in place of [1] but the arguments presented hold equally well for both forms of equation.

For ideal stratified, turbulent-turbulent and laminar-laminar flows, the values for K, a, b and c, are given in table 1.

For gas-liquid, turbulent-laminar flow, the result was

$$\left[\frac{\bar{R}_G}{\bar{R}_L}\right] = \left[W_1 \frac{\rho_G}{\rho_L} \frac{Q_G^{1.8}}{Q_L} \frac{\nu_G^{0.2}}{\nu_L D^{0.8}}\right]^{1/\omega_1} \quad [17]$$

where $\nu = \mu/\rho$ is the kinematic viscosity, D the pipe diameter and W_1 and ω_1 are constants whose values are given in table 2. For gas-liquid, laminar-turbulent flow, the result was

$$\left[\frac{\bar{R}_G}{\bar{R}_L}\right] = \left[W_2 \frac{\rho_G}{\rho_L} \frac{Q_G}{Q_L^{1\cdot 8}} \frac{\nu_G}{\nu^{0\cdot 2}} D^{0\cdot 8}\right]^{1/\omega_2} \quad [18]$$

where W_2 and ω_2 are given in table 3.

Table 1. Values of K, a, b and c in [16] for ideal stratified flow

Flow Type	Range of $\bar{R}_G/\bar{R}_L$	K	a	b	c
Turbulent-Turbulent	2.5×10^{-6} to 3×10^{-4}	1.02	0.69	0.31	0.08
Turbulent-Turbulent	3×10^{-4} to 2.1×10^{-2}	1.14	0.70	0.31	0.08
Turbulent-Turbulent	2.1×10^{-2} to 2.5×10^{-1}	1.31	0.72	0.32	0.08
Turbulent-Turbulent	2.5×10^{-1} to 1.3	1.48	0.77	0.34	0.09
Turbulent-Turbulent	1.3 to 8.0	1.49	0.83	0.37	0.09
Turbulent-Turbulent	8.0 to 1.4×10^{2}	1.28	0.90	0.40	0.10
Turbulent-Turbulent	1.4×10^{2} to 10^{5}	1.09	0.93	0.41	0.10
Laminar-Laminar	10^{-3} to 0.2	1.46	0.45	0.00	0.45
Laminar-Laminar	0.2 to 3.0	1.95	0.50	0.00	0.50
Laminar-Laminar	3.0 to 10^{3}	1.83	0.57	0.00	0.57

Table 2. Values of W_1 and ω_1 in [17] for gas-liquid turbulent-laminar stratified flow

Range of $\bar{R}_G/\bar{R}_L$	W_1	ω_1
0.1 - 0.7	0.01383	2.25
0.7 - 3.5	0.01516	2.00
3.5 - 20.0	0.01200	1.83
20.0 - 200.0	0.00826	1.70

Table 3. Values of W_2 and ω_2 in [18] for gas-liquid laminar-turbulent stratified flow

Range of $\bar{R}_G/\bar{R}_L$	W_2	ω_2
0.04 - 0.02	538.69	2.25
0.2 - 6.0	630.44	2.15
6.0 - 150.0	474.13	2.00

It should be noted that when the phases are in dissimilar flow regimes, i.e. laminar–turbulent or turbulent–laminar, the resultant equation is not of the same form as [1] or [16].

Ideal annular flow

With ideal annular flow and taking $f_i = f_G$, for turbulent–turbulent and laminar–laminar gas–liquid flow, the form of [16] was obtained also, but with the values of K, a, b and c, as given in table 4.

The solution in this case proved to be exact, consequently there was no need to resort to the use of numerical methods and curve fitting techniques since the range of $\bar{R}_G/\bar{R}_L$ was unlimited. The solutions for annular gas–liquid, turbulent–laminar and laminar–turbulent flows do not yield the same form of equation as [1] or [16] in a similar manner to that found for the case of stratified flow. Furthermore, unlike the case of stratified flow where the solutions are of practical importance, as will be discussed later, such solutions for ideal annular flow serve no practical purpose and were therefore not pursued.

The factors given by Butterworth for [1] as well as the corresponding values for [16] for the various models are given in table 5 for easy reference.

DISCUSSION AND COMPARISON WITH EXPERIMENTAL DATA

It is important to note that the analysis of idealised stratified and annular flows provides, under certain conditions, a holdup equation of the general form given by [1] or [16], and as such establishes a theoretical basis for this type of correlation. The analysis also shows that the factors appearing in [16] can be expected to vary depending, among other things, on the nature of the flow encountered in each phase, i.e. laminar or turbulent, the range of the value of $\bar{R}_G/\bar{R}_L$, and the flow pattern. Furthermore, the analysis also shows that the form of [1] or [16], is valid only when the gas and the liquid are flowing in the same regime of turbulent flow or laminar flow.

Table 4. Values of K, a, b and c in [16] for ideal annular flow

Flow type	$\bar{R}_G/\bar{R}_L$	K	a	b	c
Turbulent-Turbulent	—	1.0	0.9	0.4	0.1
Laminar-Laminar	—	1.0	0.5	0.0	0.5

Table 5. Values of A, p, q and r in [1] as given by Butterworth and the values of K, a, b and c in [16] for the various models considered

Model	A	p	q	r	K	a	b	c
Homogeneous Model	1	1	1	0	1	1	0	0
Zivi (1964)	1	1	0.67	0	1	1	0.33	0
Turner-Wallis (1965)	1	0.72	0.40	0.08	1	0.72	0.32	0.08
Lockhart-Martinelli (1949)	0.28	0.64	0.36	0.07	3.57	0.64	0.28	0.07
Thom (1962)	1	1	0.89	0.18	1	1	0.11	0.18
Baroczy (1965)	1	0.74	0.65	0.13	1	0.74	0.09	0.13
Harrison* (1975)	1	0.80	0.515	0	1	0.80	0.285	0

*Harrison (1975) obtained these values using data obtained from a 20 cm diameter geothermal steam–water pipe line.

No one set of factors given by the analytical solutions in tables 1 and 4 agree exactly with any of the models or correlations considered by Butterworth and shown in table 5. However, a number are sufficiently close to warrant some comment. The Harrison (1975) correlation shows reasonable agreement with the analytical expression developed for annular flow. This is not unexpected because, in fact, the results of steam-water annular flow of the type used by Harrison should approach the idealised turbulent annular flow case since the variation of liquid film occurring in large diameter pipes at high gas flows is small compared with the pipe size. What little variation which is observed between the two can be attributed to the inevitable presence of surface disturbances on the film, the entrained liquid droplets and the variation in film thickness in the real situation which might cause a skewing effect in the velocity profiles. In fact, in analysing the Lockhart-Martinelli relationship for pressure drop, Chen & Spedding (1981) found that data from the gas–oil, air-water and steam–water systems for large diameter pipes approached the solutions derived for ideal annular flow. The Baroczy (1965) correlation, while not being as close as the Harrison correlation, does show a similarity to the analytical expression for annular turbulent-turbulent flow. The Turner & Wallis (1965) model exhibits fair agreement with the analytical solution for turbulent-turbulent stratified flow for the condition $\bar{R}_G/\bar{R}_L < 1.0$. It is of interest to point out that while Turner & Wallis started off with a separate cylinder model, and subsequently incorporating empirical correction factors in their equation, they indicated that the range of applicability of the model was in the range of $\bar{R}_G/\bar{R}_L > 1.0$. A convenient form of equation to use for representing experimental data is that of [19]

$$\left[\frac{\bar{R}_G}{\bar{R}_L}\right] = H\left[\frac{Q_G}{Q_L}\right]^a \qquad [19]$$

which is a special form of [16] when the system conditions remain constant, i.e. when $H = K(\rho_G/\rho_L)^b(\mu_G/\mu_L)^c$. A wide range of experimental horizontal flow holdup data are accordingly plotted as $\bar{R}_G/\bar{R}_L$ vs Q_G/Q_L in figures 3, 4, 5, 6 and 7.

All the available stratified flow data were found to exhibit scatter in such a type of plot since the factors K, a, b, and c depend not only on the value of $\bar{R}_G/\bar{R}_L$, but also on whether the gas–liquid were in the laminar-laminar, laminar-turbulent, turbulent-laminar or turbulent-turbulent flow regimes. Thus it was found to be unprofitable to attempt to include stratified flow

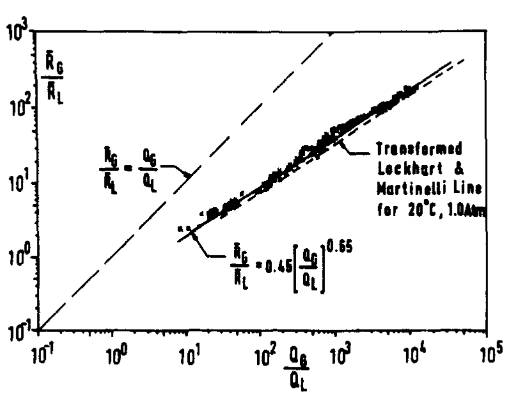

Figure 3.

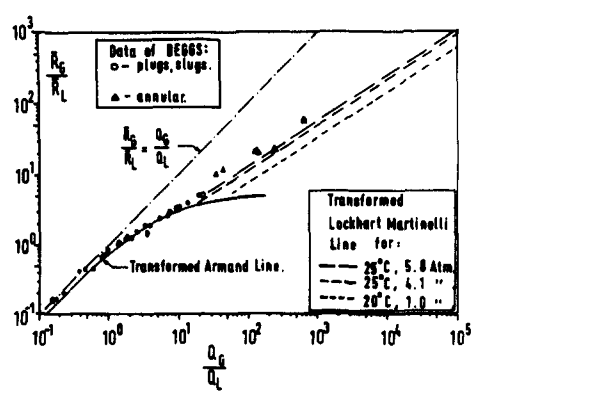

Figure 4.

Figure 3. Horizontal pipe annular flow data of Chen & Spedding (1979) compared with the transformed Lockhart-Martinelli correlation and [19].

Figure 4. Horizontal pipe, slug, plug and annular flow data of Beggs (1972) compared with the transformed Armand and Lockhart-Martinelli correlations. Note that stratified data are excluded from the graph in order to simplify the presentation.

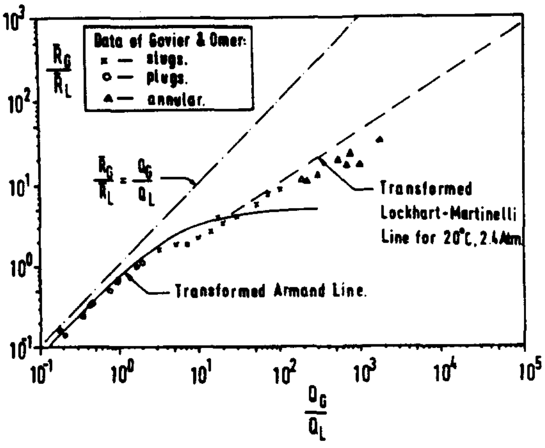

Figure 5. Horizontal pipe, slug, plug and annular flow data of Govier & Omer (1962) compared with the transformed Armand and Lockhart–Martinelli correlations.

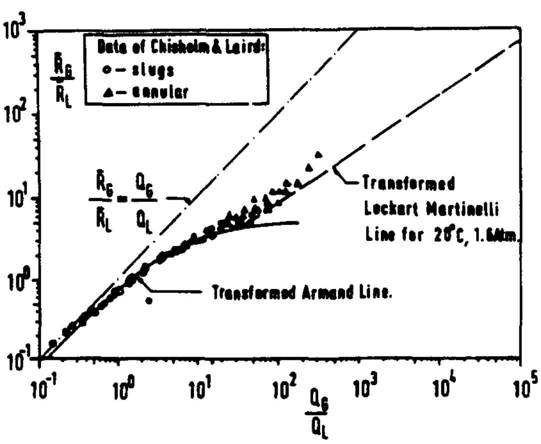

Figure 6. Horizontal pipe slug and annular flow data of Chisholm & Laird (1958) compared with the transformed Armand and Lockhart–Martinelli correlations.

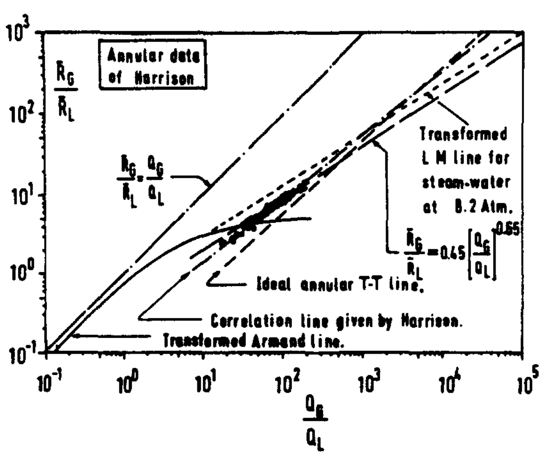

Figure 7. Horizontal pipe geothermal stream–water annular flow data of Harrison (1975) for a 20 cm dia. pipe, compared with various correlations and theory.

holdup data on this general type of plot and such data were analysed in terms of [16], [17] and [18]. Figure 8 shows the value of $\bar{R}_L$ plotted as predicted versus measured for the case of laminar liquid and turbulent gas flows. The agreement is good. The predictions of [16], (17] and [18] were further compared to the predictions of Taitel & Dukler (1976) who used a similar approach but had their solutions expressed in terms of the Lockhart-Martinelli $\bar{R}$-X parameters. Figure 9 shows the comparison, also for the case of laminar liquid and turbulent gas flow, and the agreement is very good. A more detailed comparison is given in Spedding & Chen (1979b). Equations [16], [17] and [18] have also been applied successfully to the determination of flow pattern transitions by Spedding & Chen (1981) to describe the stratified-annular, the stratified-slug and the annular-slug transitions.

All the data for the slug and plug flow pattern fall about a curved line in the range of $(\bar{R}_G/\bar{R}_L) < 4.0$, as evidenced in figures 4, 5 and 6. It was found that this curved line may be obtained by expressing the empirical equation of Armand (1946), $\bar{R}_G = 0.83[Q_G/(Q_G + Q_L)]$, in terms of $(\bar{R}_G/\bar{R}_L)$ and (Q_G/Q_L), thus,

$$\left[\frac{\bar{R}_G}{\bar{R}_L}\right] = \frac{1}{[0.2 + 1.2/(Q_G/Q_L)]}. \qquad [20]$$

In terms of dryness fraction, Armand's equation is

$$\left[\frac{\bar{R}_L}{\bar{R}_G}\right] = 1.2\left[\frac{1-x}{x}\right]\left[\frac{\rho_G}{\rho_L}\right] + 0.2. \qquad [21]$$

Comparing [21] with [1], and referring to table 5, leads to the conclusion that the equation of

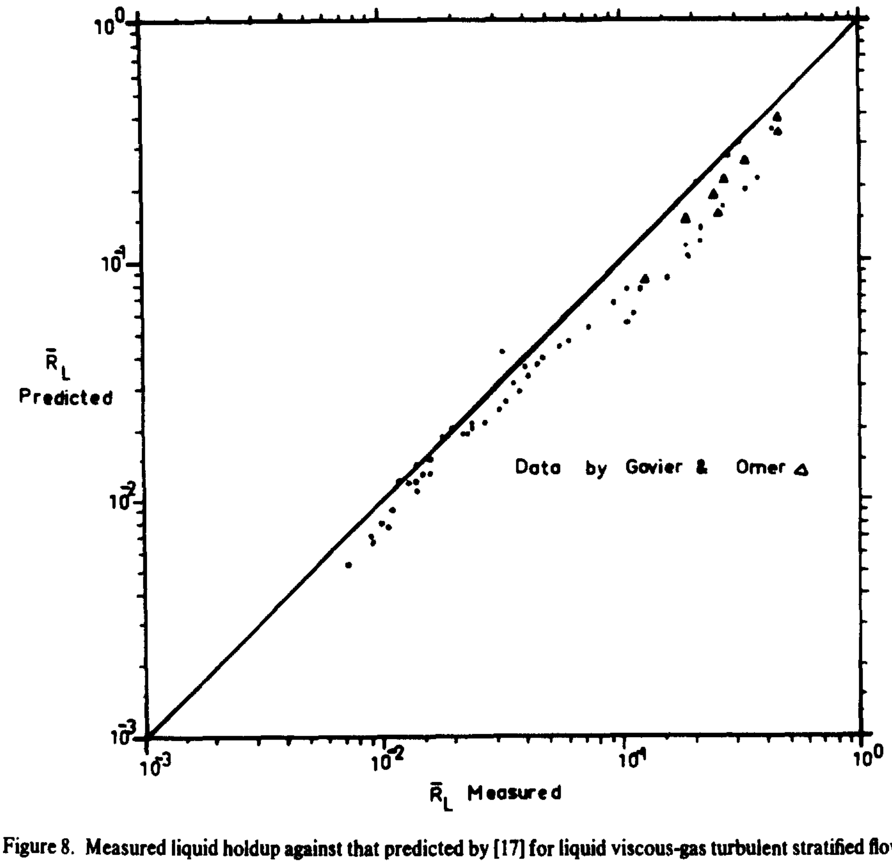

Figure 8. Measured liquid holdup against that predicted by [17] for liquid viscous-gas turbulent stratified flow.

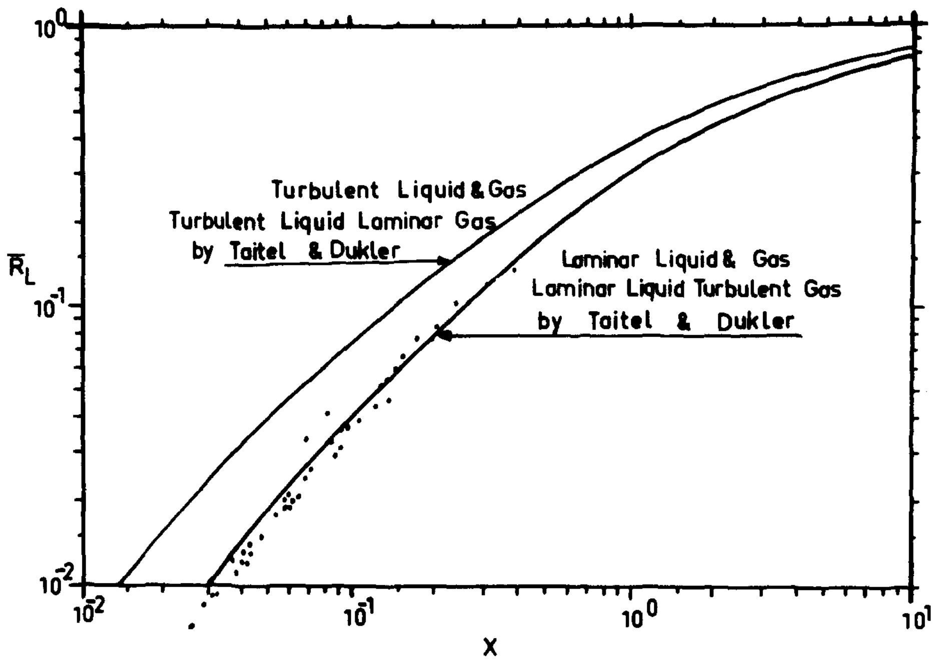

Figure 9. Comparison of the prediction using [17] and the $\bar{R}_L - X$ lines of Taitel & Dukler (1976) for liquid viscous-gas turbulent stratified flow together with data of Chen & Spedding (1979).

Armand also is a special form of Butterworth's equation [1], but with an additional term of 0.2 added.

Nguyen & Spedding (1977) developed a holdup equation which can be reduced to the more simple Zuber & Findlay (1965) form under certain circumstances

$$\frac{\bar{V}_{SG}}{\bar{R}_G} = C_0 \bar{V}_T + B' \qquad [22]$$

where $\bar{V}_{SG}$ is the superficial gas velocity based on the total cross-sectional area of the pipe and $\bar{V}_T$ is the total superficial velocity of gas and liquid, C_0 is the distribution parameter which accounts for non-uniformity of flow, while B' is the initial function and is related to the mean velocity difference between the phases. Rearrangement gives

$$\left[\frac{\bar{R}_L}{\bar{R}_G}\right] = C_0\left[\frac{1-x}{x}\right]\left[\frac{\rho_G}{\rho_L}\right] + (C_0 - 1) + \frac{B'}{\bar{V}_{SG}} \qquad [23]$$

which again is a modified form of [1] to which two terms have been added. Taking $C_0 = 1.2$ as a rough approximation, as suggested by Zuber & Findlay (1965), it is obvious that [23] and [21] are basically the same except for the term $B'/\bar{V}_{SG}$. They are both, in fact, modified forms of the homogeneous model and hence of [1] but with the important difference that they do not apply to the idealised annular or stratified flow regimes, above $\bar{R}_G/\bar{R}_L = 4.0$, since the mixture region is non-existent, but must be combined with another correlation of the form of [19] if generality is to be obtained.

A further model which may be cast in the Butterworth form, is the equal velocity head model due to Smith (1971). However the analysis, while being claimed to be quite general, leads to a complex result in which the indices vary progressively between wide limits set by the cases for ideal annular flow and ideal homogeneous misty flow.

In figure 3 the data of Chen & Spedding (179) for annular flow are shown to obey [19]. Also included in the plot is the transformed Lockhart-Martinelli line as given by Butterworth with the appropriate constants given in table 5 for 20°C and 1.0 atm pressure. As expected for the case of small diameter pipes, the Lockhart-Martinelli holdup correlation over-predicts $\bar{R}_L$. The ideal annular flow equations with factors given in table 4, however, deviate significantly from the data, which are adequately represented if $K = 0.45$ and $a = 0.65$ in [19]. For these conditions this is the form of correlation which, applying above $\bar{R}_G/\bar{R}_L = 4.0$, allows generality to be achieved.

In figure 4 the slug and plug, and annular flow data of Beggs (1972) are plotted. Also included is the transformed Armand equation [20], and the Lockhart-Martinelli equation for 20°C, 1 atm; 25°C, 4.1 atm; 25°C, 5.8 atm. It is obvious that the slug and plug flow data obey the Armand equation while the Lockhart-Martinelli equation generally overpredicts $\bar{R}_L$. The slug, plug and annular flow of Govier & Omer (1962), shown in figure (5), generally agree with the Armand equation also, while the Lockhart-Martinelli equation for 20°C and 2.4 atm is shown to underpredict $\bar{R}_L$. The slug and annular flow data of Chisholm & Laird (1958) are given in figure 6, together with Armand's line and the Lockhart-Martinelli equation for 20°C and 1.6 atm. Again the slug and plug flow data obey the Armand equation, with the annular flow data showing deviations which are consistent with those observed in figure 4.

Data of Harrison (1975) for geothermal steam-water flow is plotted in figure 7 together with the transformed Lockhart-Martinelli line, the turbulent-turbulent annular flow line and the correlation suggested by Harrison for the steam-water condition at 8.2 atm. The data points are enveloped by the Lockhart-Martinelli line and the ideal annular flow line. The fact that they fall about the same line which represents the annular air-water flow data of Chen & Spedding (1979) may be coincidental since the two different systems also are completely different in terms of pipe diameter, pressure and temperature.

It is observed that in the system variables covered by the various sets of data considered, slug and plug flow data obey the Armand equation satisfactorily independent of system conditions. However, the Armand equation has been reported to vary at conditions vastly different from those of the atmospheric air-water system as outlined by Chisholm (1973), and Bankoff (1960), among others. Thus the effects of viscosity, mass velocity, etc. will require to be determined if such variation was envisaged.

For annular flow, the transformed Lockhart-Martinelli equation overpredicts $\bar{R}_L$ for small diameter pipes but underpredicts for larger pipes; a circumstance which is possibly linked to the fact that the data used for small diameter pipes were those for the air-water system, and data used for larger diameter pipes were those for steam-water. Nevertheless, it appears that the experimental data and the prediction by the transformed Lockhart-Martinelli equation are generally consistent since they both vary directly with the parameter $(Q_G/Q_L)^a$. Thus, it is suggested that [19] is a convenient form of equation for data presentation for annular flow in the case of fixed system conditions. While the value of H, as shown by comparing [19] to [16], appeared to be a function only of (ρ_G/ρ_L) and (μ_G/μ_L), it is suggested that a further variable, namely the pipe size, must also be incorporated into the correlation. The effect of mass velocity on the void fraction in annular flow has been detailed by Zuber & Findlay (1965).

Chen & Spedding (1981) have extennded the Lockhart-Martinelli theory to enable pressure drop and holdup to be calculated for separated flow. The derived relationships for holdup are quite satisfactory for the case of stratified flow but exhibit a large deviation for annular flow data. Such a deviation between experimental and theory also has been noted in this work. The reasons for the disagreement were suggested as being due to variation of film thickness around the pipe, the skewing of the velocity profile, entrained drops and the effect of interfacial disturbances. Since the effect appears to be magnified in small diameter pipes it is possible that the first two possible mechanisms only are involved under such geometrical circumstances. when ideality is more closely approached in large diameter pipes the film thickness and velocity profile effects tend to become of minor importance. But, as has been already pointed out, that reasonable agreement between theory and experiment is achieved even in 20 cm diameter pipes used by Harrison (1975) for the steam–water system. The most realistic explanation of the lack of agreement appears to lie with the interfacial phenomena involved in the practical situation. In the theoretical development already presented it was assumed that the interface was smooth and hence $f_i \simeq f_G$. Such an assumption is closer to the real situation in the case of stratified flow and hence the reasonable agreement with experimental result as reported in Spedding & Chen (1979). However, in the case of annular flow large deviations occur because $f_i \neq f_G$ due to the presence of ripples and roll waves (Wallis 1970). While f_L may be approximated using the liquid film Reynolds number and the Blasius form of equation, the evaluation of f_i is more difficult, and the correlation of Wallis (1970) as given in [15] is substituted into [14] to give

$$\bar{R}_L = \frac{1}{1 + \left[\dfrac{0.005(1+75\bar{R}_L)}{f_L(1-\bar{R}_L)^{1/2}}\right]^{1/2}\left[\dfrac{\rho_G}{\rho_L}\right]^{1/2}\left[\dfrac{Q_G}{Q_L}\right]}. \qquad [24]$$

The annular flow holdup data of Chen & Spedding (1979), Govier & Omer (1962) and Chisholm & Laird (1958) were used to check the validity of [24] in figure 10 in which the $\bar{R}_L$ values are plotted as predicted against actual. Reasonable agreement is achieved despite the fact that the values of $\bar{R}_L$ in annular flow are small, i.e. $\bar{R}_L < 0.2$, so that any error in such a log–log correlation will be greatly amplified. Thus [24] is suggested as an accurate correlation for the determination of liquid holdup in annular horizontal two-phase flow.

In order to obtain $\bar{R}_L$ form [24], an iterative process must be used. Even so, the process is quite simple and speedy because the equation converges very rapidly. However, the calculation may be performed more conveniently by the use of computer techniques.

It is also of interest to consider the slip ratio S which is defined as the ratio of the true

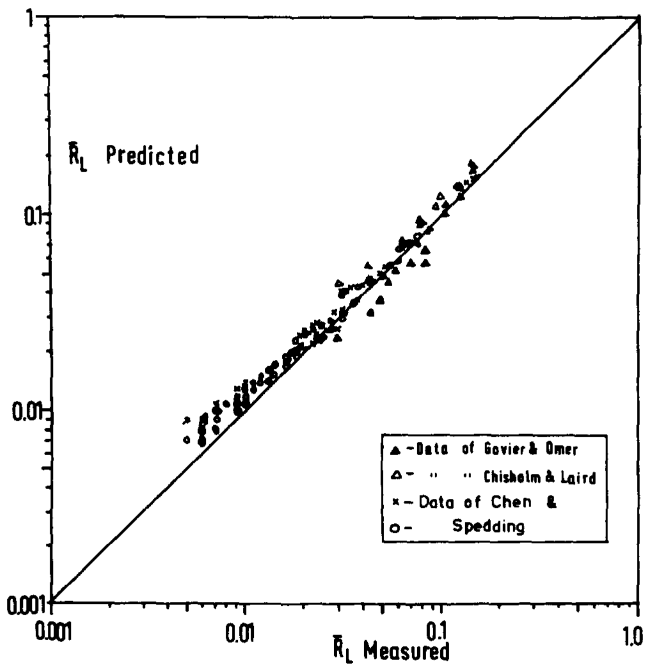

Figure 10. Predicted $\bar{R}_L$ using [24] against measured $\bar{R}_L$ for horizontal annular flow from various sources.

velocities. Expressing in terms of the Q and $\bar{R}$ values

$$S = \frac{Q_G}{\bar{R}_G}\frac{\bar{R}_L}{Q_L}. \qquad [25]$$

Equation [25] gives, for Harrison's goethermal steam–water data of $Q_G/Q_L = 10$ to 200, a value of $S = 5$ to 14.

An equation given by Chisholm (1973) gives

$$S = \left[\frac{1 + \frac{Q_G}{Q_L}}{1 + \frac{\rho_G}{\rho_L}\frac{Q_G}{Q_L}}\right]^{1/2} \qquad [26]$$

which, for the geothermal steam–water system of Harrison with $\rho_G/\rho_L \simeq 0.003$, gives values of $S = 3$–12. This is in agreement with the results presented in this work.

For the air–water system, the value of S ranges between 1.0 and 50. Using [26] and with $\rho_G/\rho_L \simeq 0.0013$, the values of S is 1–27, showing that [26] under-predicts S in the high flow rate air–water system. However, [26] is strictly only applicable to $Q_G/(Q_L + Q_G) < 0.9$, and that the flow must not be stratified. It was observed that at the upper range of Q_G/Q_L (Chen & Spedding 1979) the annular film breaks down to result in a stratified flow with only a trickle of liquid running along the bottom of the pipe, and thus there is insufficient liquid to wet the entire pipe perimeter.

CONCLUSIONS

An analytical solution of idealised models has given some theoretical justification to the Butterworth form of holdup correlation, as given by [1] or [16], when the two phases are flowing in the same flow regime, i.e. both laminar or both turbulent. It was found that the factors A, p, q and r in the case of [1], or K, a, b and c in the case of [16], varied depending on the values of $(\bar{R}_L/\bar{R}_G)$, the flow regime, and the flow pattern. However, when the gas–liquid were in dissimilar flow regimes, i.e. laminar–turbulent or turbulent–laminar, the holdup equation derived did not follow the form of [1] or [16].

As suggested by the form of [16] and [19] experimental data were plotted as $(\bar{R}_G/\bar{R}_L)$ against (Q_G/Q_L) from which it was found that holdup data may be broadly classified into three major groups based on the flow pattern; stratified, annular, and slugs and plugs. The method given by Spedding & Chen (1981), which makes use of the stratified holdup equations given here, is useful for determining these groupings. Stratified flow data are represented by the stratified flow equations derived in this work and which also are reported in Spedding & Chen (1979b). Annular flow data is best represented by [16] or its simplified form, [19], although some diameter and mass velocity effects are evident. Slug and plug flow data are described best by the Armand type of equation, or its transform as given by [20], for system whose properties are close to those of the atmospheric air–water system. By using an experimentally derived form for the interfacial friction factor it is possible to derive a correlation for liquid holdup for annular flow which gives reasonable agreement with a wide range of data and thus appears to be more general in scope than [16] or [19].

REFERENCES

ARMAND, A. A. 1946 The resistance during the movement of a two-phase system in horizontal pipes. *Izv. VTI* 1, 16–23, 1946 *AERE Trans.* 828.

BANKOFF, S. G. 1960 A variable density single fluid model for two-phase flow with particular reference to steam–water flow. *Trans. ASME, J. Heat Transfer* 265–272.

BAROCZY, C. J. 1965 Correlation of liquid fraction in two-phase flow with application to liquid metals. *A.I.Ch.E. CEP Symp. Series* **61**, 179–191.

BEGGS, H. D. 1972 An experimental study of two-phase flow in inclined pipes. Ph.D. Dissertation, Univ. Tulsa.

BERGELIN, O. P. & GAZLEY, C. 1949 Co-current gas–liquid flow. I. Flow in horizontal tubes. *Heat Transfer and Fluid Mech. Inst., Calif. Meet., ASME.*

BUTTERWORTH, D. 1972 Air–water annular flow in a horizontal tube. *Prog. Heat Mass Transfer* **6**, 235–251.

BUTTERWORTH, D. 1975 A comparison of some void-fraction relationships for gas–liquid flow. *Int. J. Multiphase Flow* **1**, 845–850.

CHEN, J. J. J. & SPEDDING, P. L. 1979 Data on holdup, pressure loss and flow pattern in a horizontal pipe. *Univ. Auckland, Engng Report* 214.

CHEN, J. J. J. & SPEDDING, P. L. 1981 An extension of the Lockhart–Martinelli theory of two-phase pressure drop and holdup. *Int. J. Multiphase Flow*, **7**, 659–675.

CHISHOLM, D. 1973 Research Note: Void fraction during two-phase flow. *J. Mech. Engng Sci.* **15**, 235–236.

CHISHOLM, D. & LAIRD, A. D. K. 1958 Two-phase flow in rough pipes. ASME paper 57-SA-11.

GOVIER, G. W. & OMER, M. M. 1962 The horizontal pipeline flow of air–water mixtures. *Can. J. Chem. Engng* 93–104.

HARRISON, R. F. 1975 Methods for the analysis of geothermal two-phase flow. M.E. Thesis. Univ. of Auckland.

LOCKHART R. W. & MARTINELLI, R. C. 1949 Proposed correlation of data for isothermal two-phase two-component flows in pipes. *Chem. Engng Prog.* **45**, 39–48.

NGUYEN, V. T. & SPEDDING, P. L. 1977 Holdup in two-phase gas liquid flow—I. Theoretical aspects. *Chem. Engng Sci.* **32**, 1003–1014.

SMITH, S. L. 1971 Void fractions in two-phase flow: A correlation based upon an equal velocity head model. *Heat Fluid Flow* **1**, 22–39.

SPEDDING, P. L. & CHEN, J. J. J. 1979a Correlation of holdup in two-phase flow. *ANZAAS*, **49**, Auckland.

SPEDDING, P. L. & CHEN, J. J. J. 1979b Correlation and estimation of holdup in two-phase flow. *Proc. N. Z. Geothermal Workshop* **1**, 180–199.

SPEDDING, P. L. & CHEN, J. J. J. 1981 A simplified method of determining flow pattern transition of two-phase flow in a horizontal pipe. *Int. J. Multiphase Flow* **7**, 729–731.

TAITEL, Y. & DUKLER, A. E. 1976 A theoretical approach to the Lockhart–Martinelli correlation for stratified flow. *Int. J. Multiphase Flow* **2**, 591–595.

THOM, J. R. S. 1962 Prediction of pressure drop during forced circulation boiling of water. *Int. J. Heat Mass Transfer* **7**, 709–724.

TURNER, J. M. & WALLIS, G. B. 1965 The separate-cylinder model of two-phase flow. Paper No. NYO-3114-6. Thayer School of Engng, Dartmouth College.

WALLIS, G. B. 1970 Annular two-phase flow. Part I. A simple theory. Part II. Additional effects. *Trans. ASME, J. Basic Engng* 59–72; 73–81.

ZIVI, S. M. 1964 Estimation of steady-state steam void fraction by means of the principle of minimum entropy production. *Trans. ASME J. Heat Transfer* **86C**, 247–252.

ZUBER, N & FINDLAY, J. A. 1965 Average volumetric concentration in two-phase flow systems. *Trans. ASME J. Heat Transfer* **87C**, 453–468.